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Measurement of Fluid Resistance Correction Factor for a Sphere Moving through a Viscous Fluid toward a Plane Surface

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When the Reynolds number of the flow around the sphere is much less than 1.0, the fluid resistance is evaluated from the Stokes' formula:

$$F = 6\pi\mu r_p u_p$$

where r_p is the radius of a sphere, u_p is the sphere velocity, and μ is the fluid viscosity. This equation was obtained by neglecting the inertial terms in the Navier-Stokes equation for a rigid sphere in an unbounded fluid. Therefore, this relation applies only to fluid media which extend to infinity in all directions. In fact, when a sphere vertically approaches the rigid walls and/or a free surface, its velocity decreases as a result of an increase in the drag caused by additional friction between the fluid and the plane surface. Brenner (1961), therefore, calculated the correction factor to modify the form of Stokes' equation, solving the equation of creeping motion by use of bipolar coordinates.

Mackay and Mason (1961) measured the trajectories of single spheres approaching to the glass plate by using a 16-mm cine camera fitted with a 2.5-mm lens. Mackay et al. (1963) compared these experimental data with the theoretical values by Brenner. In this study, we experimentally examined the decreases in the velocity of single spheres in two cases that a sphere is approaching to the solid surface and the free surface, and the experimental results which are converted in the form of drag are compared in detail with the theoretical value given by Brenner.

The objective of this paper is to show that when a sphere approaches the vicinity of a plane surface (except the region where the London-Van der Waals attractive force is predominant), the drag of a sphere comes to about 50 or more times as large as that in an unbounded fluid and that the effect of increase in the drag should be considered when one analyzes the collection mechanism of particle such as in air and water filtration because the particle slowly moves toward the surface of the collector to be captured in those cases.

EXPERIMENT

When a sphere slowly approaches a plane surface with a velocity u_p in an unbounded and quiescent fluid, the form of Stokes' formula should be modified with the correction factor β calculated by Brenner (1961):

$$F = 6\pi\mu r_p \beta u_p \qquad (1)$$

where β is a function of the separation h between the sphere and the plane surface only.

By use of Eq. 1, the equation of motion of a sedimentary sphere in the coordinate system as taken in Figure 1 is:

$$m_p\left(1 - \frac{\rho_f}{\rho_p}\right)\frac{du_p}{dt} = -m_p\left(1 - \frac{\rho_f}{\rho_p}\right)g - 6\pi\mu r_p \beta u_p \qquad (2)$$

where ρ_p and ρ_f are the densities of the sphere and the fluid respectively; g is gravitational acceleration; m_p is the mass of a sphere; and t is time.

β is given from Eq. 2:

$$\beta = -\frac{2}{9}\cdot\frac{r_p^2 g}{\mu}\rho_p\left(1 - \frac{\rho_f}{\rho_p}\right)\cdot\left(1 + \frac{du_p}{dt}\frac{1}{g}\right)\Big/u_p \qquad (3)$$

On the other hand, the terminal sedimentation or rise velocity u_∞ in the Stokes region is:

$$u_\infty = -\frac{2}{9}\cdot\frac{r_p^2 g}{\mu}\rho_p\left(1 - \frac{\rho_f}{\rho_p}\right) \qquad (4)$$

Accordingly, Eq. 3 is expressed as:

$$\beta = \frac{u_\infty}{u_p}\left(1 + \frac{du_p}{dt}\cdot\frac{1}{g}\right) \qquad (5)$$

Considering that $|du_p/dt| << 1$, the final form of β is:

$$\beta = \frac{u_\infty}{u_p} \qquad (6)$$

This equation means that if u_p and u_∞ are experimentally evaluated, the value of β at an arbitrarily given position can be known.

Solid Surface

The schematic diagram of experimental setup is illustrated in Figure 2. An acrylic table of 100-mm height was laid at the bottom of a transparent acrylic vessel of $100 \times 100 \times 710 mm^3$ as is seen in Figure 2. After millet jelly whose density ($\rho = 1.185 - 1.421$ g/cm^3) and viscosity ($\mu = 0.698 - 47.8$ g/cm·s) had already been known was poured into the vessel and the homogeneity of its concentration was confirmed, a glass sphere ($r_p = 1.245$cm, $\rho_p = 2.500$g/cm^3) was gently dropped into it.

The sedimentation velocity was examined as follows. After recording the figure of the sedimentary sphere on the videotape through television camera with a ring for close-up photograph using an electronic flash (i.e., stroboscope), the videotape was slowly rerecorded on a monitor television. Its magnification was 24.0. The positions of bottom apex of the sphere were marked at on-and-off intervals of the electronic flash which could be known by the sudden shining of the photos on the monitor television.

Dividing the each distance s between two marks ($s \fallingdotseq 1/600\ r_p$, when the sphere was very near to the wall; $s = 0.07\ r_p$, when the sphere was far apart from the wall) by the flash time interval, the sphere velocity at each position was estimated. The rotation number of the stroboscope used was 300-500 rpm examined by

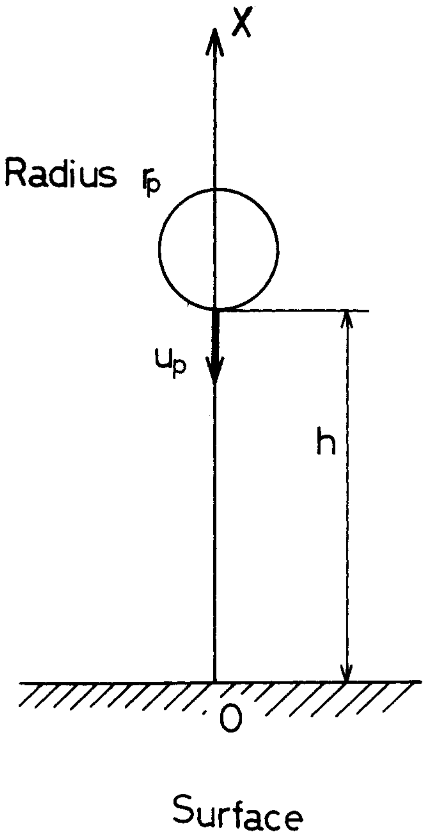

Figure 1. Co-ordinates.

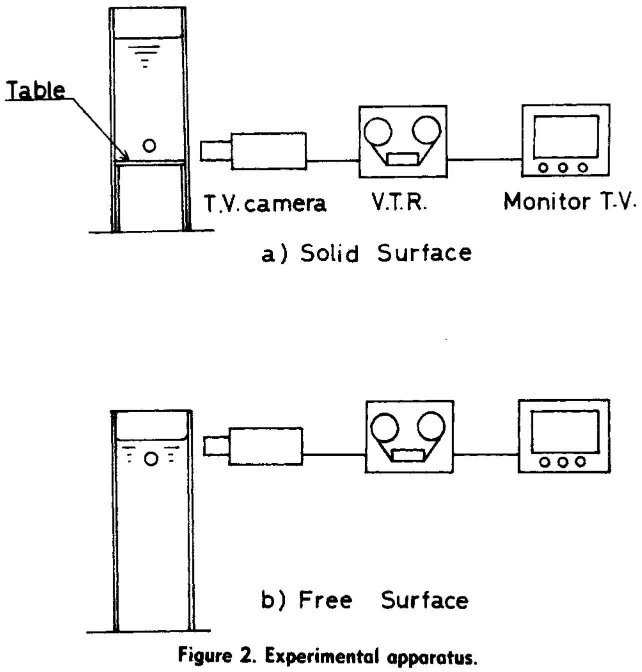

Figure 2. Experimental apparatus.

use of the universal counter (Takeda Riken Kogyo). To determine u_∞, the sedimentation velocity in the region far apart from both the bottom wall and the top free surface (i. e., the region further than 20 r_p from both surfaces in our case) was examined. Substitution of u_p and u_∞ determined by the preceding method into Eq. 6 yields the value of β at an arbitrarily given position.

The temperature change of millet jelly during a run to measure u_p and u_∞ was 0.5°C at highest and so its effect on the experimental result is negligibly small.

To determine the viscosity of millet jelly, the terminal sedimentation velocity of the small steel sphere (r_p = 0.150cm, ρ_p = 7.837 g/cm^3) dropped gently into the millet jelly was examined in the same way as noted above. Because the steel sphere is so small that the presence of the side walls does not affect its terminal velocity. The viscosity of millet jelly was evaluated by substituting the terminal velocity of the steel sphere into Stokes' equation. (Needless to say, the Reynolds number on the sphere basis is much less than 1.) The density of millet jelly was examined by use of a hydrometer.

Free Surface

An acrylic sphere (r_p = 1.25cm, ρ_p = 1.18 g/cm^3) was buoyed up through the millet jelly in the same vessel to determine the rise velocities u_p and u_∞, similar to that described in "**Solid Surface.**" To determine the viscosity of millet jelly this way, a polycarbonate sphere (r_p = 0.156cm, ρ_p = 1,200g/cm^3) was used.

RESULTS AND DISCUSSION

The experimental results for solid surface and free surface are plotted in Figures 3 and 4, respectively. Sudden increase in the value of β is observed around the region where the separation h comes to about 0.1 r_p for both cases. In fact, the value of β reaches about 50 for solid and about 10 for free surfaces, respectively. This means that the drag on the sphere comes to about 50 or about 10 times as much as that in an unbounded fluid. The solid line in the figures indicates the theoretical value by Brenner (1961). Though the experimental values are spread, the experimental and theoretical values are in fairly good agreement, which may justify the Brenner's theory experimentally.

According to the theory, however, the infinitely large force is required for the particle adhesion to the wall because the theoretical value of β at $h = 0$ exhibits infinitely large. Although this seems to contradict the actual phenomenon, one may reasonably understand it by considering the action of London-Van der Waals force, Fw, which predominates when h is much smaller than $0.01r_p$. In fact, the value of Fw increases in proportion to h^{-4}, while the value of β does in proportion to h^{-1}. Accordingly, when h is very small, the sphere can easily collide with the wall.

In our experimental range ($h > 0.01r_p$), the effect of Fw on u_p is negligibly small, because $Fw/6\pi\mu r_p u_\infty$ is of the order of 10^{-6}. In conclusion, we present the experimental evidence to support that the application of the Brenner's theory is quite reasonable to the drag on the sphere in the fluid which can be regarded as continuum.

The main reasons for the spread and the deviation of experimental results are:

1. Since the value of u_∞ is experimentally underestimated by the presence of the top and bottom surfaces (i. e., the free and solid surfaces), the value of β is also underestimated as is obvious from Eq. 6.
2. The value of u_∞ is underestimated by the presence of side walls.
3. Although the Reynolds number is supposed to be nearly equal to zero, this number in our experiment was of the order of 10^{-1}.
4. Since the distance measured on the monitor television to determine the sphere velocity in the vicinity of the free or solid surface was as small as about 2mm, the large experimental error unavoidably arises.

According to the Brenner's theoretical equation, the decrease in the value of u_∞ by the presence of the wall is about 6% at $h = 20r_p$. Since the value of u_∞ was determined at the position which is about $20r_p$ far from the top and bottom surfaces, the total error caused by the presence of the solid and free surfaces comes to about 10%. Hence, the error caused by the reason 1. is estimated at about 10%.

Considering the sphere size (2.5cm) and the width of the vessel (10cm), the effect of the side walls on the decrease in the

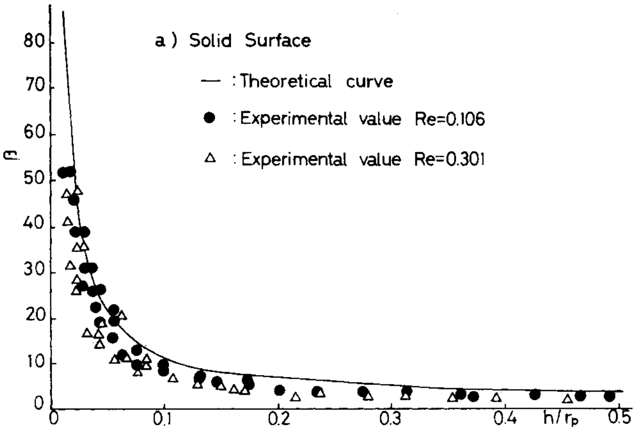

Figure 3. β **vs.** h/r_p **(solid surface).**

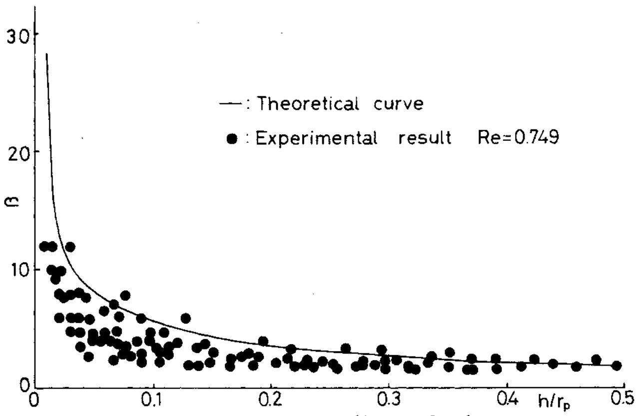

Figure 4. β **vs.** h/r_p **(free surface).**

value of u_∞ is not negligible. Its decrease in the value of u_∞ is estimated at about 32% in our condition according to Faxen's equation (1922). However, the value of u_p is also equally affected by the presence of the side walls. Therefore, the value of β is not affected much by the presence of the side walls, because the errors included in the denominator u_p and the numerator u_∞ of Eq. 6 cancel out each other. These separate effects cannot be linearly superposed (Sonshine et al., 1966). This causes some discrepancies between observed and calculated results.

It is certain that the experimental result is affected by the Reynolds number, because the increase in this number causes the experimental values to deviate considerably from the theoretical curve.

The preceding discussion on the reasons 1., 2., and 3. explains the fact that the experimental values are smaller than the theoretical ones. The spread of the experimental values is applied to the reason 4.

The experimental values in the case of the free surface spread more than those in the case of the solid surface. This is so, because it was very difficult to keep the concentration of millet jelly (i.e., the density of millet jelly) in the vicinity of the free surface homogeneous.

The comparison of Figures 3 and 4 indicates that the value of β in the case of the free surface is smaller than that in the case of the solid surface at the same h. There might be two reasons for this. First, the tangential stresses vanish on the free surface. Secondly, when the sphere approaches the vicinity of the free surface, the free surface is raised by the push of the sphere. The latter was not considered in Brenner's analysis.

NOTATION

F	= resistance force
F_w	= London-Van der Waals force
h	= minimum separation between sphere and plane surfaces
g	= gravitation acceleration
m_p	= mass of sphere
r_p	= radius of sphere
s	= distance between two marks
t	= time
u_p	= sphere velocity
u_∞	= Stokes sedimentation or rise velocity
β	= correction factor
ρ_f, ρ_p	= densities of fluid and sphere
μ	= fluid viscosity

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Correlation of Eddy Diffusivities for Pipe Flow

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The design of heat and mass transfer equipment requires an accurate knowledge of heat and mass transfer Nusselt numbers. These can be theoretically predicted and correlated from the turbulent transport equations, if one knows how the eddy diffusivities vary with Reynolds' number, fluid properties and the distance from the wall. Such knowledge would also be useful in the design of tubular reactors in which radial transport is important (Berker and Whitaker, 1978; Sundaram and Froment, 1979).

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